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# RELATION BETWEEN THE BDMPS TRANSPORT COEFFICIENT AND THE DIPOLE CROSS SECTION

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**Abstract**
In the BDMPS formalism, the transverse momentum accumulated by a quark propagating through nuclear matter is proportional to the so-called transport coefficient $\hat{q}$. On the other hand, transverse momentum broadening can also be calculated within the color dipole approach where it is expressed in terms of the dipole cross section $\sigma_{q\bar{q}}$. Since both approaches are equivalent, it is possible to find a relation between $\hat{q}$ and $\sigma_{q\bar{q}}$.

Transverse momentum broadening of a fast parton (*i.e.* quark or gluon) propagating through nuclear matter has received much attention during the past years, because it is intimately related to the medium induced energy loss of that parton [1, 2]. Indeed, it was found by the BDMPS collaboration [1] that in QCD the induced radiative energy loss per unit length is proportional to the transverse momentum acquired by the parton. This leads to the seemingly counterintuitive result $-dE/dz \propto L$, where $L$ is the length of the medium traversed by the parton. The same result was obtained by Zakharov [3] in the color dipole approach. Even though these two approaches use very different languages to describe induced energy loss, it was found in [4] that they are equivalent. In this note, we present a quantitative relation between the two main non-perturbative inputs of both approaches, the BDMPS transport coefficient $\hat{q}$ and the dipole cross section $\sigma_{q\bar{q}}$, Eq. (5). To our knowledge, this relation was first published in [5].

Broadening of transverse momentum of a fast quark propagating in nuclear matter (but not energy loss) was first investigated within the dipole approach in [6]. The authors of [6] study broadening of Drell-Yan pairs in hadron-nucleus collisions. The result of this phenomenological analysis is that the projectile quark performs a random walk as it propagates through the nucleus and thereby acquires the transverse momentum

$$\delta\langle p_T^2\rangle = 2C(\widetilde{x},\widetilde{Q}^2)\rho_A L, \tag{1}$$

where $\rho_A$ is the nuclear density. The factor $C(\widetilde{x},\widetilde{Q}^2)$ originates from the dipole cross section, which can be written for small separations $\rho$ as

$$\sigma_{q\bar{q}}(\widetilde{x},\rho) = C(\widetilde{x},\rho)\rho^2. \tag{2}$$

A theoretically more profound derivation of Eq. (1) can be found in [5, 7]. Note that $C$ depends on energy $\widetilde{x}$ and on a hard scale $\widetilde{Q}^2$. At leading order, these scales cannot be calculated exactly and one has to rely on plausible arguments to find their values, see [7]. The coefficient $C$ can be expressed in terms of the gluon density $G_N$ of a nucleon [8],

$$C(\widetilde{x},\rho) = \frac{\pi^2}{3}\alpha_s(\lambda/\rho^2)\widetilde{x}G_N(\widetilde{x},\lambda/\rho^2), \tag{3}$$

where $\lambda$ is a dimensionless number. Note that in order to obtain broadening for gluons, one would have to multiple $C$ by the ratio of Casimir factors for adjoint and fundamental representation of $SU(3)$, *i.e.* $C_A/C_F = 9/4$.

On the other hand, for cold nuclear matter the BDMPS transport coefficient $\hat{q}$ can also be expressed in terms of the gluon density of a nucleon [1]. For the quark case we are interested in, one finds

$$\hat{q} = \frac{2\pi^2\alpha_s(Q'^2)}{3}\rho_A x' G_N(x',Q'^2). \tag{4}$$

Again, the scales $x'$ and $Q'^2$ cannot be determined exactly. The transport coefficient describes the "scattering power" of the medium. Our final result is obtained by comparing Eqs. (3) and (4), which yields

$$\hat{q}(x', Q'^2) = 2\rho_A C(\widetilde{x}, \widetilde{Q}^2). \tag{5}$$

This equality holds to logarithmic accuracy because of the scale dependence of $\hat{q}$ and $C$. In addition, one finds from Eqs. (1), (3) and (4)

$$\delta\langle p_T^2\rangle = \hat{q}L = p_{\perp W}^2, \tag{6}$$

where (following the notation of [1]) $p_{\perp W}^2$ is the characteristic transverse momentum squared of a parton produced in the medium after traversing nuclear matter of length $L$. The second equality in Eq. (6) was found within the BDMPS formalism in [1].

We stress that Eq. (5) was not obtained by equating Eq. (1) with the corresponding result from the BDMPS approach, *i.e.* $p_{\perp W}^2 = \hat{q}L$. Instead, we have shown that $\delta\langle p_T^2\rangle = p_{\perp W}^2$, confirming the equivalence of the two approaches. Note that it is not clear a priori that these two quantities should be equal, since $\delta\langle p_T^2\rangle$ is the broadening of an incident quark, while $p_{\perp W}^2$ is the broadening for a quark produced in the medium. However, since both quantities are proportional to the length of the medium, the result $\delta\langle p_T^2\rangle = p_{\perp W}^2$ is plausible.

As a concluding remark, we mention that one could attempt to set $\delta\langle p_T^2\rangle$ equal to the nuclear broadening obtained in the higher twist factorization approach of [9]. However, even though the relation between BDMPS/color dipole formalism and LQS approach is not fully understood yet, the LQS approach probably contains different physics and is not equivalent to the other two approaches. A more detailed discussion can be found in [1, 5]. Therefore, the relation between the LQS parameter $\lambda_{LQS}$ [9] and the gluon density such a procedure would yield [1, 5] should be interpreted with great care. The connection between LQS and BDMPS/color dipole approach certainly needs further study.

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## References

[1] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B **484** (1997) 265 [arXiv:hep-ph/9608322].

[2] F. Arleo, these proceedings (and references therein).

[3] B. G. Zakharov, JETP Lett. **65** (1997) 615 [arXiv:hep-ph/9704255].

[4] R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, Nucl. Phys. B **531** (1998) 403 [arXiv:hep-ph/9804212].

[5] U. A. Wiedemann, Nucl. Phys. A **690** (2001) 731 [arXiv:hep-ph/0008241].

[6] J. Dolejši, J. Hüfner and B. Z. Kopeliovich, Phys. Lett. B **312** (1993) 235 [arXiv:hep-ph/9305238].

[7] M. B. Johnson, B. Z. Kopeliovich and A. V. Tarasov, Phys. Rev. C **63** (2001) 035203 [arXiv:hep-ph/0006326].

[8] B. Blättel, G. Baym, L. L. Frankfurt and M. Strikman, Phys. Rev. Lett. **70** (1993) 896; L. Frankfurt, A. Radyushkin and M. Strikman, Phys. Rev. D **55** (1997) 98 [arXiv:hep-ph/9610274].

[9] M. Luo, J. w. Qiu and G. Sterman, Phys. Rev. D **49** (1994) 4493.